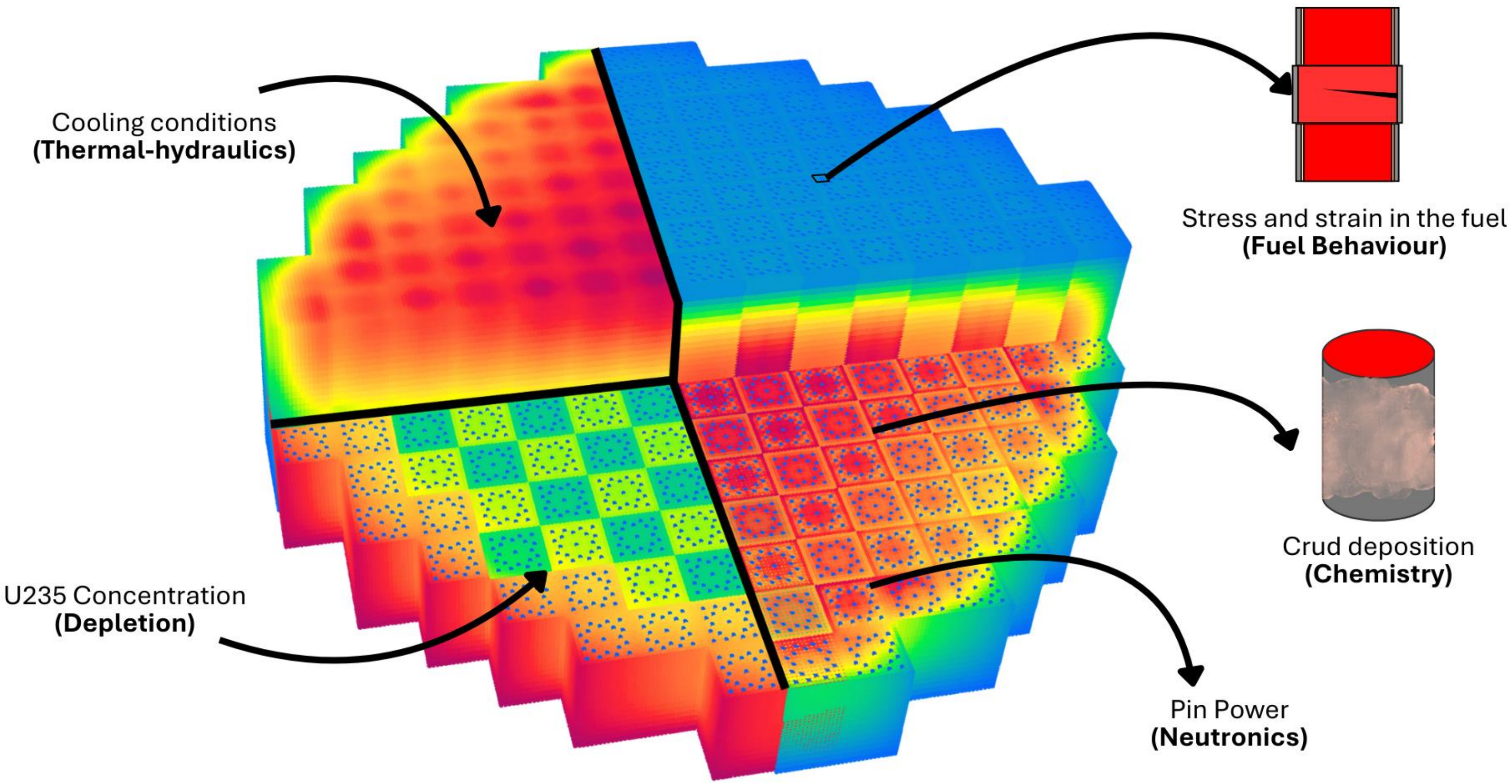


# Physics of Nuclear Reactors

## Kinetics Part I

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# Exercises of today

**12 NEUTRON KINETICS – PART 1**

12.1 Prompt neutron lifetime
12.2 Prompt neutron lifetime II
12.3 Stable period T proportional to 1/ρ
12.4 Stable period I

***Divide in groups of 5:***

- *We will do Exercise 1 together at the board*
- *I will leave you time for Exercise 2 (10 minutes)*
- *We will go through the solution together*
- *I will leave you time for Exercise 3 (10 minutes)*
- *We will go through the solution together*

# 1. Prompt neutron lifetime



**Exercise description:**

Calculate the prompt neutron lifetime in an infinite critical thermal reactor, composed of a homogeneous mixture of $^{235}U$ and $H_2O$ of unit density at ambient temperature.

The diffusion time $t_{d,m}$ in $H_2O$ is 2.1.10$^{-4}$ s and $\eta_{th}$=2.065.

**Knowledge to be applied:** $k_\infty = \eta_{\text{th}} f = 1$

**Expected results:** $l = 1.1 \times 10^{-4} s$

EPFL

# 1. Prompt neutron lifetime

**Take a step back, the prompt kinetics equations:**

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \bar{\nu}F(t) - (A(t) + L(t))$$

Evolution of the neutron population

Production by fission

Loss by absorption

Loss by leakage

**Recasting the equation using k:**

$$k_{\text{eff}}(t) = \frac{\bar{\nu}F(t)}{A(t) + L(t)} \implies \frac{\mathrm{d}P}{\mathrm{d}t} = (k_{\text{eff}}(t) - 1) \cdot (A(t) + L(t))$$

**Assuming proportionality of loss term to neutron population:**

$$[A, L] \propto P \implies A(t) + L(t) = cst \cdot P(t) \simeq \frac{P(t)}{l}$$

Measure of the time taken for the disappearance of neutrons

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{k_{\text{eff}}(t) - 1}{l} P(t)$$

**Solution**

$$P(t) = P(0) \exp\left(\frac{k_{\text{eff}} - 1}{l} t\right)$$

**Does it remind you of something?**

EPFL

# 1. Prompt neutron lifetime

**Take a step back, what is $l$:**

**Remember how flux and neutron density relate?**

- One has: $l = \dfrac{P(t)}{A(t)+L(t)}$ with $A(t) = \int_V \Sigma_a \phi(\vec{r},t)\mathrm{d}V = \Sigma_a v \int_V n(\boldsymbol{r},t)\mathrm{d}V = \Sigma_a v P(t)$
- Leakage ~ supplementary absorptions corresponding to: $\Sigma_a' \simeq DB^2$ → **Remember the reactor equation?** $\nabla^2\phi + B_m^2\phi = 0$
- Thus, $L(t) = DB^2 v P(t) \implies A(t) + L(t) = (\Sigma_a + DB^2)vP(t)$
- i.e. $l = \dfrac{P(t)}{A(t)+L(t)} = \dfrac{1}{v(\Sigma_a + DB^2)} = \dfrac{1}{v\Sigma_a(1+B^2L^2)}$

**You need to express this as a function of:**
moderator diffusion time and fuel utilization factor

# 1. Prompt neutron lifetime



**What is the geometrical buckling for an infinite thermal reactor?**

For an infinite thermal reactor ($B^2 = 0$).

**What is the prompt neutron lifetime for an infinite system?**

$$l = t_d$$ → **Diffusion time**

**How can we relate this to the moderator diffusion time?**

$$l = t_d = \frac{1}{v\Sigma_a} = \frac{1}{v\Sigma_{a,m}}\frac{\Sigma_{a,m}}{\Sigma_a} = \frac{\lambda_{a,m}}{v}(1-f) = t_{d,m}(1-f)$$ → **Remember four factor formula?**

where $t_{d,m} = \frac{\lambda_{a,m}}{v}$ is the diffusion time in the moderator.

Because the reactor is critical, $k_\infty = \eta_{th} f = 1$, i.e. $f = \frac{1}{\eta_{th}} = \frac{1}{2.065} = 0.484$

Thus, $\boldsymbol{l = t_{d,m}(1-f) = 2.1 \times 10^{-4}(1 - 0.484) = 1.1 \times 10^{-4} s}$

# 2. Prompt neutron lifetime II



**Exercise description:**

Calculate the values of l for an infinite critical thermal system, consisting of a homogeneous mixture of $^{233}U$ and

(a) $D_2O$ (with 0.25% $H_2O$ impurity), $t_{dm}=4.3.10^{-2}$ s

(b) Be , $t_{dm}=3.9.10^{-3}$ s

(c) graphite, $t_{dm}=0.017$ s

where $t_{dm}$ is the diffusion time in the moderator.

For $^{233}U$ , $\eta_{th}=2.285$.

**Knowledge to be applied:** $t_{d,m} = \frac{\lambda_{a,m}}{v}$, $l = \frac{1}{v\Sigma_a}$, $k_\infty = \eta_{\text{th}} f = 1$, $\frac{\Sigma_{a,m}}{\Sigma_a} = (1-f)$

**Expected results:** (a) $l = 2.4 \times 10^{-2} s$ (b) $l = 2.2 \times 10^{-3} s$ (c) $l = 0.010 s$

**Exercise solution:**

# 2. Prompt neutron lifetime II



As in Ex. 12.1, $l = t_{d,m}(1 - f)$ with $f = \frac{1}{\eta_{\text{th}}}$

For $^{233}U$, $\eta_{th}$=2.285 , so that f = 0.438.
Thus,

a) $l = 4.3 \times 10^{-2}(1 - 0.438) = 2.4 \times 10^{-2}s$

b) $l = 3.9 \times 10^{-3}(1 - 0.438) = 2.2 \times 10^{-3}s$

c) $l = 0.017(1 - 0.438) = 0.010s$

# 3. Stable period T proportional to 1/ρ

**Exercise description:**

Show that for small values of reactivity, the stable period T is inversely proportional to ρ. What is the order of magnitude of the values of T for which this relationship is valid?

**Knowledge to be applied:** $|\omega_i| << \lambda_i \Rightarrow \rho \simeq \Lambda\omega_1 + \sum_{i=1}^{6}\left[\frac{\beta_i\omega_1}{\lambda_i}\right], T = \frac{1}{\omega_1}$

**Expected results:** $T \propto \frac{1}{\rho}$, condition of validity ($|\omega_i| << \lambda_i$) $T >> 80s$

# 3. Stable period T proportional to 1/ρ



## Take a step back, the point kinetics equations with delayed neutrons:

- Fraction $\beta$ **of n's in reactor** are **delayed**, so that the neutron production rate $\neq \bar{\nu}F$
- It is, in fact: $\underbrace{\bar{\nu} \cdot F \cdot (1-\beta)}_{\text{Prompt Sources}} + \underbrace{\sum_{i=1}^{6} \left[ \underbrace{\lambda_i}_{\text{Decay constant}} \times \underbrace{C_i(t)}_{\text{Precursor density at time t}} \right]}_{\text{Delayed Sources}}$
- Thus $$\frac{dP}{dt} = \bar{\nu} F(t)(1-\beta) - [A(t)+L(t)] + \sum_{i=1}^{6} \lambda_i C_i(t)$$
- As before, substituting

$$k_{\text{eff}}(t) = \frac{\bar{\nu} F(t)}{A(t)+L(t)} \qquad \ell = \frac{P(t)}{A(t)+L(t)}$$

$$\Rightarrow \frac{dP}{dt} = \frac{(1-\beta)k_{\text{eff}}(t) - 1}{\ell} \times P(t) + \sum_{i=1}^{6} \lambda_i C_i(t)$$

Introducing the concept of reactivity

$$\rho = \frac{k_{eff} - 1}{k_{eff}}$$

$$\Rightarrow \frac{dP}{dt} = \frac{\rho(t) - \beta}{\Lambda} P(t) + \sum_{i=1}^{6} \lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} + \lambda_i C_i(t) = \frac{\beta_i}{\Lambda} P(t), \quad i = 1,\ldots 6$$

**A few ingredients to solve these:**

- Laplace transform
- Partial fraction decomposition
- Laplace transform again

# 3. Stable period T proportional to 1/ρ



**The point kinetics equations with delayed neutrons has the following solution :**

$$P(t) = \sum_{i=1}^{7} B_j \exp(\omega_j t) \quad \omega_j \text{ solutions of } \rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{(\omega + \lambda_i)}$$

| Group | Precursor | T1/2 (s) | λi (1/s) | βi (%) |
|---|---|---|---|---|
| 1 | $^{87}Br$, $^{142}Cs$ | 55.7 | 0.012 | 0.022 |
| 2 | $^{137}I$, $^{88}Br$ | 22.7 | 0.031 | 0.142 |
| 3 | $^{138}I$, $^{89}Br$, … | 6.2 | 0.11 | 0.127 |
| 4 | $^{139}I$, Cs, … | 2.3 | 0.30 | 0.257 |
| 5 | $^{140}I$, Kr, … | 0.61 | 1.14 | 0.075 |
| 6 | Br, Rb, … | 0.23 | 3.01 | 0.027 |

For sufficiently small values of ρ, such that $|\omega_i| << \lambda_i$, the **Reactivity Equation** simplifies as follows:

$$\rho \simeq \Lambda\omega + \sum_{i=1}^{6} \frac{\beta_i \omega}{\lambda_i} = \omega\left(\Lambda + \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i}\right)$$

**Thus,** $T = \frac{1}{\omega} \simeq \frac{1}{\rho}\left(\Lambda + \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i}\right)$**… (1) , i.e** $T \propto \frac{1}{\rho}$

☑ **Proof done, what is the condition of validity.**

**The condition of validity* is** $|\omega| \ll \lambda_i$ **i.e.** $|\omega| \ll \lambda_1$**,** $\lambda_1$ **being the smallest** $\lambda_i$ value (0.0124 $s^{-1}$ in LWRs). This corresponds to values of T , for which: $T = \frac{1}{|\omega|} \gg \frac{1}{\lambda_1}$, i.e. $T$>>80$s$